The Willows School

Calculation Policy

An academy within:

nexus Multi Academy Trust

"Learning together, to be the best we can be"

Mathematics is a highly creative and connected skill that is fundamental to everyday life. We learn skills that will help us function and achieve as an adult. A high quality mathematics education provides us with the foundations for understanding the world around us and the ability to reason mathematically. Lessons should be met with a sense of enjoyment and create a willingness to learn and curiosity about the subject.

Introduction

This Calculation Policy has been produced in line with the National Curriculum for Mathematics to ensure consistency and progression in teaching throughout the school.

It aims to introduce children to the processes of calculation through concrete, pictorial and abstract activities. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases and learn to interpret and use the signs and symbols involved. This policy shows the natural progression that a child can make in their mathematical education. Children should not progress onto the advanced stages of formal written methods until they have a secure conceptual understanding. This process can take many forms, a fluid approach should be adopted, and is not age related at the Willows School. Students will progress relative to themselves and each learning journey is a personal one at The Willows School.

Intent
Maths is a journey and long-term goal, achieved through exploration, clarification, practice and application over time. At each stage of learning, children should be able to demonstrate a conceptual understanding of the topic and be able to build on this over time. • Maths is taught to help our students develop the skills needed for life. • Develop a broad range of skills that students can use and apply. • Basic skills- addition, subtraction, multiplication, division, time and money rotated throughout the year, embedding learning. • Applied lessons/game based lessons lets our students use the skills they have learned. • Topic lesson are taught using real world scenarios with no abstract concepts. • Have a high quality maths curriculum that is both challenging and enjoyable, and builds upon previous learning. • We promote independence in maths, a passion for maths, having a go and 'trying our best'. • Students should "want" to learn and not feel like that they "have" to.

Implementation

- All plans and methodology are consistent throughout school enabling support and facilitating challenge.
- Maths lessons are well resourced throughout school.
- We use "varied repetition." Basic skills are repeated but the lessons are varied.
- We encourage cross curricular links and applied learning.
- Local visits are encouraged and applied learning further develops mathematics
- Active use of ICT and games to further enhance learning
- Interventions and 1 to 1 support for students who find certain areas tricky to grasp
- Maths should always be engaging and relevant.

This policy is designed to help parents, carers and other family members support children's learning by providing an explanation of the methods we use in our school. The policy is set out in the 4 operations, the basic skills we use throughout school, addition, subtraction, multiplication and division. Within each specific area there is a progression of skills, knowledge and layout for written methods. The calculation strategies which will be used will reflect this ideology – moving from concrete to pictorial and then abstract recording leading to more formal written methods. Mental methods and strategies will work in partnership with these methods. A variety of mental calculation methods will be taught throughout school. The progression of mental methods and expectations will comply with the National Curriculum Statements. At The Willows School it is important that staff always use correct subject specific mathematical language and encourage this from every pupil. This mathematical language may be individual words or phrases which when used by the pupil will develop confidence and hopefully embed knowledge and skills. This will take place in class discussions, applied learning, community based learning as well as through oral and written feedback, next steps and target setting.

"Depth in early learning is much more important than covering lots of things in a superficial way."

Failure to adopt this approach to learning could potentially lead to misconceptions and poor mathematical foundations and eventually, in later years, pupils will not be able to make any progress.

Concrete, pictorial and Abstract (CPA), The Concrete Pictorial Abstract (CPA) approach is highly effective in the teaching of Maths to develop conceptual understanding. This approach will vary between each pupil and each individual ability.
Manipulatives (objects), pictorial representations, words, numbers and symbols are everywhere. Our approach incorporates all of these to help children explore and demonstrate mathematical ideas, enrich their learning experience and deepen understanding. Together, these elements give our students the best chance of grasping knowledge and skills so they truly understand what they've learnt.
All pupils, when introduced to a key new concept, should have the opportunity to build competency in this topic by taking this approach. Pupils are encouraged to physically represent mathematical concepts. Concrete objects and resources (manipulatives) that allow students to explore an idea in an active, hands-on approach and pictures are used to demonstrate and visualise abstract ideas, alongside numbers and symbols.

<u>Concrete</u> – The doing stage. There is a clear focus on the use of concrete objects and visual images to support understanding for every student. Each new concept or calculation strategy will be introduced using appropriate concrete objects, giving the children a clear picture of the theoretical mathematics they are learning. It is important that children have access to a wide range of concrete objects in every class and, consequently, we encourage children to be independent in their use of concrete objects throughout the school and access resources as they see fit. We can achieve this through the effective use of and plentiful of resources around school This is the foundation for conceptual understanding.

Concrete resources that may be found in classrooms will include:





These resources will vary depending on year group and individual needs. At home, pupils very well may not have access to these school resources; however, they are just a vehicle to support a pupil's understanding of a topic. Any objects, such as stones, sticks, toy cars can be used at home to replace counters, cubes etc.

<u>Pictorial</u> <u>– The seeing stage</u> - A child has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or a picture of the problem.

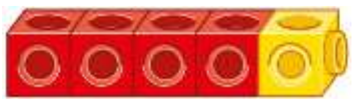
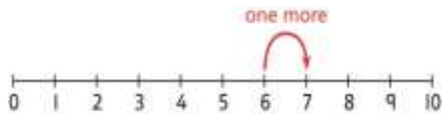
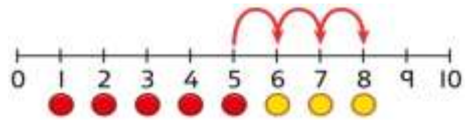
<u>Abstract</u><u>- The symbolic stage</u> - A child is now capable of representing problems by using mathematical notation, for example 10 ÷ 2 = 5

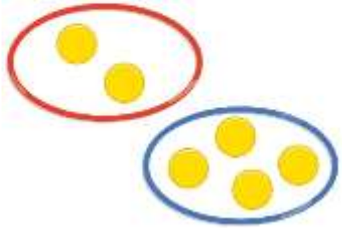
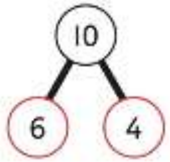
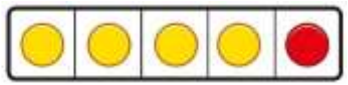
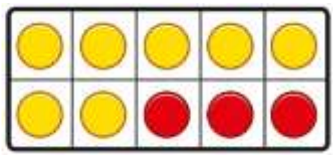
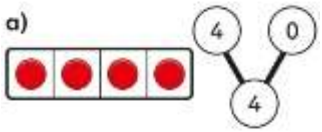
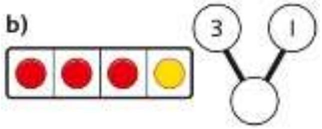
Impact

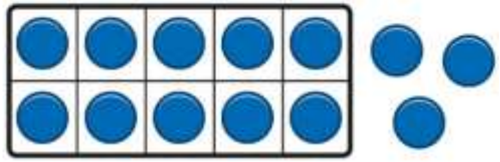
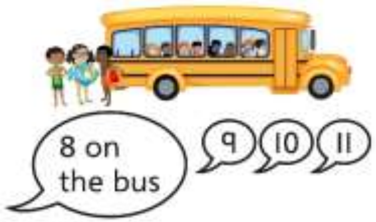
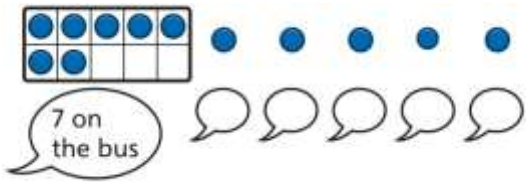
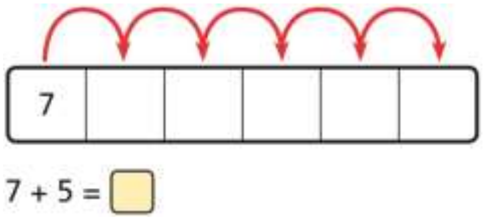
Pupils will leave us prepared for the next stage in their lives with:

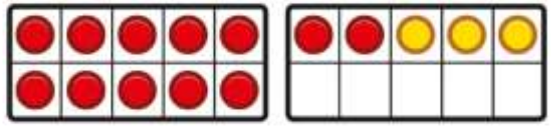
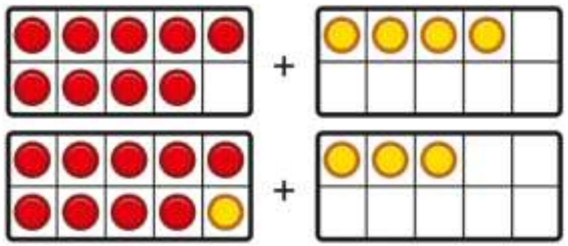
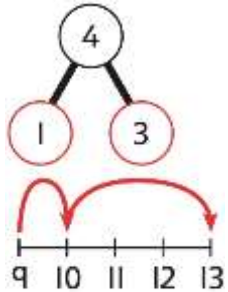
- The ability to "use" mathematics outside of school. "Learning for life and Preparing for Adulthood"
- Varied qualification paths that suit everybody.
- No students should ever be "scared" of maths
- Students will take what they learn and be able to use it and apply it everywhere.

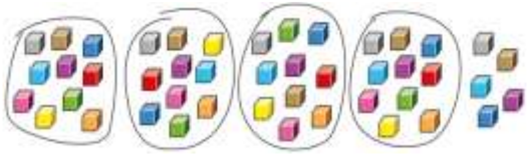
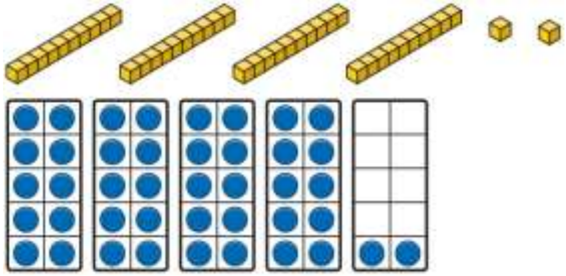
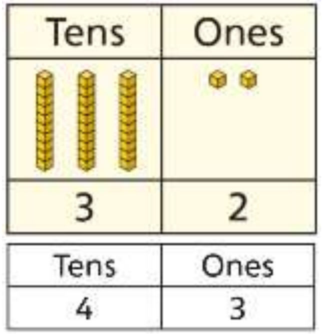
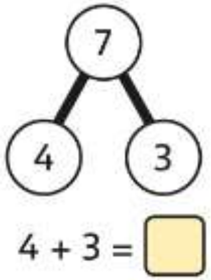
- A knowledge and/or understanding of facts and procedures.
- The potential to move between different contexts and representations of mathematics
- The ability to recognise relationships and make connections in mathematics
- Confidence and belief that they can achieve
- The knowledge that maths underpins most of our daily lives
- Have a positive and inquisitive attitude to mathematics as an interesting and attractive subject in which all children can achieve

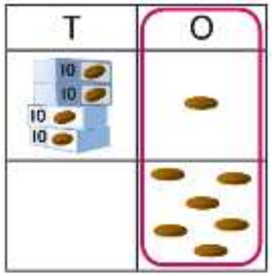
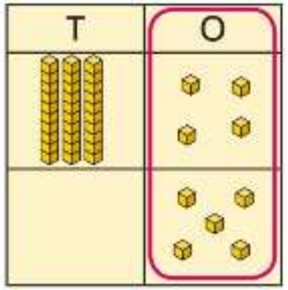
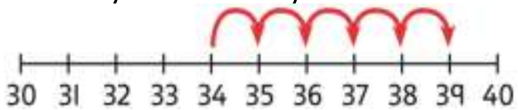
Addition			
	Concrete	Pictorial	Abstract
Counting and adding more	Children add one more person or object to a group to find one more.	Children add one more cube or counter to a group to represent one more.  *One more than 4 is 5.*	Use a number line to understand how to link counting on with finding one more.  *One more than 6 is 7.* *7 is one more than 6.* Learn to link counting on with adding more than one.  *5 + 3 = 8*

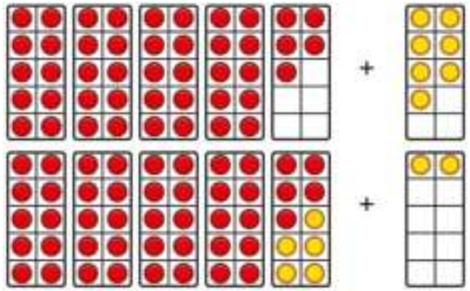
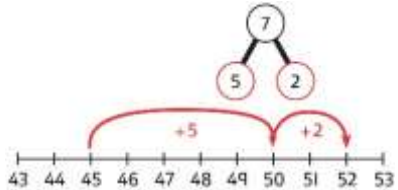
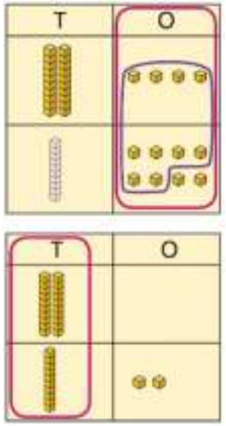
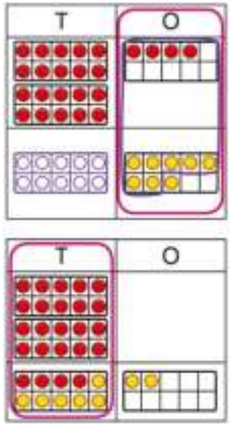
Understanding part-part-whole relationship	Sort people and objects into parts and understand the relationship with the whole.  *The parts are 2 and 4. The whole is 6.*	Children draw to represent the parts and understand the relationship with the whole.  *The parts are 1 and 5. The whole is 6.*	Use a part-whole model to represent the numbers.  $\boxed{6} + \boxed{4} = \boxed{10}$ *6 + 4 = 10*
Knowing and finding number bonds within 10	Break apart a group and put back together to find and form number bonds.  *3 + 4 = 7*  *6 = 2 + 4*	Use five and ten frames to represent key number bonds.  *5 = 4 + 1*  *10 = 7 + 3*	Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero. a)  b)  *4 + 0 = 4* *3 + 1 = 4*

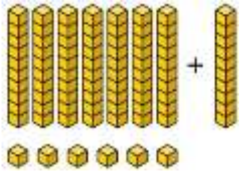
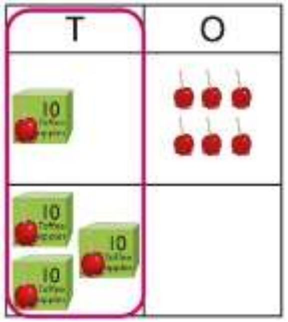
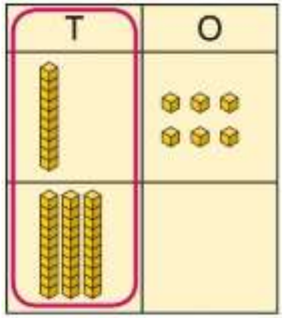
Understanding teen numbers as a complete 10 and some more	Complete a group of 10 objects and count more.  *13 is 10 and 3 more.*	Use a ten frame to support understanding of a complete 10 for teen numbers.  *13 is 10 and 3 more.*	*1 ten and 3 ones equal 13.* *10 + 3 = 13*
Adding by counting on	Children use knowledge of counting to 20 to find a total by counting on using people or objects. 	Children use counters to support and represent their counting on strategy. 	Children use number lines or number tracks to support their counting on strategy. 

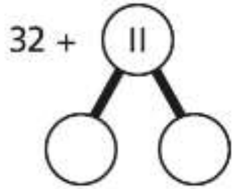
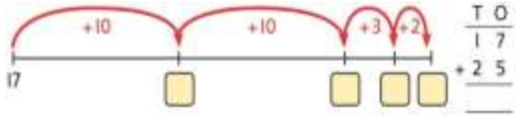
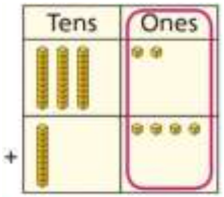
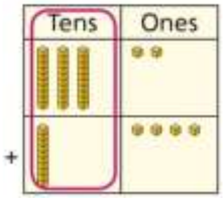
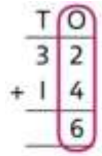
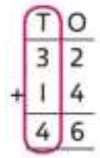
Adding the 1s	Children use bead strings to recognise how to add the 1s to find the total efficiently.  *2 + 3 = 5* *12 + 3 = 15*	Children represent calculations using ten frames to add a teen and 1s.  *2 + 3 = 5* *12 + 3 = 15*	Children recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently. *3 + 5 = 8* *So, 13 + 5 = 18*
Bridging the 10 using number bonds	Children use a bead string to complete a 10 and understand how this relates to the addition.  *7 add 3 makes 10.* *So, 7 add 5 is 10 and 2 more.*	Children use counters to complete a ten frame and understand how they can add using knowledge of number bonds to 10.  .	Use a part-whole model and a number line to support the calculation.  *9 + 4 = 13*

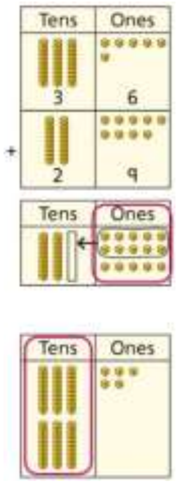
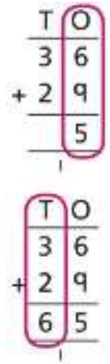
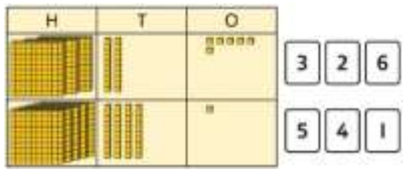
Understanding 10s and 1s	Group objects into 10s and 1s.  Bundle straws to understand unitising of 10s. 	Understand 10s and 1s equipment, and link with visual representations on ten frames. 	Represent numbers on a place value grid, using equipment or numerals. 
Adding 10s	Use known bonds and unitising to add 10s.  *I know that 4 + 3 = 7.* *So, I know that 4 tens add 3 tens is 7 tens.*	Use known bonds and unitising to add 10s.  *I know that 4 + 3 = 7.* *So, I know that 4 tens add 3 tens is 7 tens.*	Use known bonds and unitising to add 10s.  *4 + 3 = 7* *4 tens + 3 tens = 7 tens* *40 + 30 = 70*

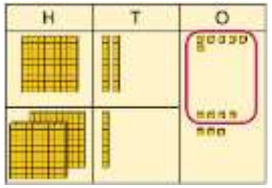
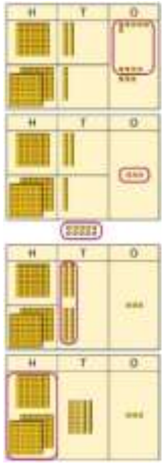
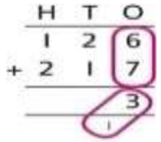
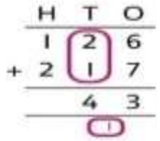
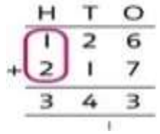
Adding a 1-digit number to a 2-digit number not bridging a 10	Add the 1s to find the total. Use known bonds within 10.  *41 is 4 tens and 1 one.* *41 add 6 ones is 4 tens and 7 ones.* This can also be done in a place value grid.  	Add the 1s.  *34 is 3 tens and 4 ones.* *4 ones and 5 ones are 9 ones.* *The total is 3 tens and 9 ones.*  	Add the 1s. Understand the link between counting on and using known number facts. Children should be encouraged to use known number bonds to improve efficiency and accuracy.  This can be represented horizontally or vertically. *34 + 5 = 39* *or* T O 3 4 + 5 9

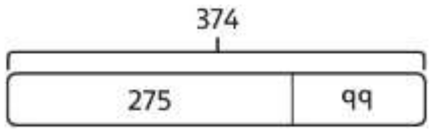
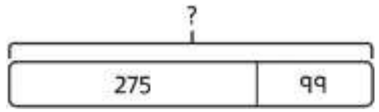
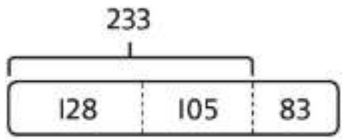
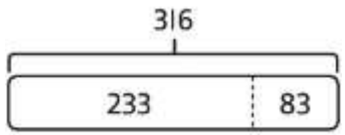
Adding a 1-digit number to a 2-digit number bridging 10	Complete a 10 using number bonds.  *There are 4 tens and 5 ones. I need to add 7. I will use 5 to complete a 10, then add 2 more.*	Complete a 10 using number bonds.  	Complete a 10 using number bonds.  *7 = 5 + 2* *45 + 5 + 2 = 52*
Adding a 1-digit number to a 2-digit number using exchange	Exchange 10 ones for 1 ten.  	Exchange 10 ones for 1 ten.  	Exchange 10 ones for 1 ten. 

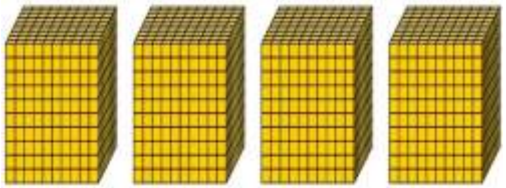
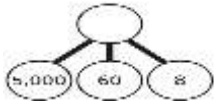
Adding a multiple of 10 to a 2-digit number	Add the 10s and then recombine.  *27 is 2 tens and 7 ones.* *50 is 5 tens.* *There are 7 tens in total and 7 ones.* *So, 27 + 50 is 7 tens and 7 ones.*	Add the 10s and then recombine.  *66 is 6 tens and 6 ones.* *66 + 10 = 76* A 100 square can support this understanding. 	Add the 10s and then recombine. *37 + 20 = ?* *30 + 20 = 50* *50 + 7 = 57* *37 + 20 = 57*
Adding a multiple of 10 to a 2-digit number using columns	Add the 10s using a place value grid to support.  *16 is 1 ten and 6 ones.* *30 is 3 tens.* *There are 4 tens and 6 ones in total.*	Add the 10s using a place value grid to support.  *16 is 1 ten and 6 ones.* *30 is 3 tens.* *There are 4 tens and 6 ones in total.*	Add the 10s represented vertically. Children must understand how the method relates to unitising of 10s and place value. $\begin{array}{cc} T & O \\ \hline 1 & 6 \\ +\,3 & 0 \\ \hline 4 & 6 \end{array}$ *1 + 3 = 4* *1 ten + 3 tens = 4 tens* *16 + 30 = 46*

Adding two 2-digit numbers	Add the 10s and 1s separately.  *5 + 3 = 8* *There are 8 ones in total.* *3 + 2 = 5* *There are 5 tens in total.* *35 + 23 = 58*	Add the 10s and 1s separately. Use a part-whole model to support.  *11 = 10 + 1* *32 + 10 = 42* *42 + 1 = 43* *32 + 11 = 43*	Add the 10s and the 1s separately, bridging 10s where required. A number line can support the calculations.  *17 + 25*
Adding two 2-digit numbers using a place value grid	Add the 1s. Then add the 10s.  		Add the 1s. Then add the 10s.  

Adding two 2-digit numbers with exchange	Add the 1s. Exchange 10 ones for a ten. Then add the 10s. 		Add the 1s. Exchange 10 ones for a ten. Then add the 10s. 
3-digit number + 3-digit number, no exchange	Use place value equipment to make a representation of a calculation. This may or may not be structured in a place value grid. *326 + 541 is represented as:* 	Represent the place value grid with equipment to model the stages of column addition.	Use a column method to solve efficiently, using known bonds. Children must understand how this relates to place value at every stage of the calculation.

3-digit number + 3-digit number, exchange required	Use place value equipment to enact the exchange required.  *There are 13 ones.* *I will exchange 10 ones for 1 ten.*	Model the stages of column addition using place value equipment on a place value grid. 	Use column addition, ensuring understanding of place value at every stage of the calculation.    *126 + 217 = 343* Note: Children should also study examples where exchange is required in more than one column, for example *185 + 318 = ?*

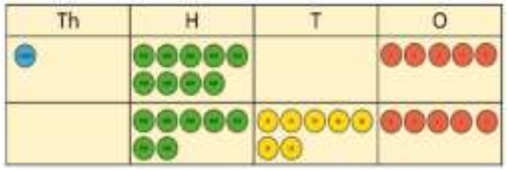
Representing addition problems, and selecting appropriate methods	Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods.	Children understand and create bar models to represent addition problems. *275 + 99 = ?*  *275 + 99 = 374*	Use representations to support choices of appropriate methods.  *I will add 100, then subtract 1 to find the solution.* *128 + 105 + 83 = ?* *I need to add three numbers.* 128 + 105 = 233  

Understanding numbers to 10,000	Use place value equipment to understand the place value of 4-digit numbers.  *4 thousands equal 4,000.* *1 thousand is 10 hundreds.*	Represent numbers using place value counters once children understand the relationship between 1,000s and 100s.  *2,000 + 500 + 40 + 2 = 2,542*	Understand partitioning of 4-digit numbers, including numbers with digits of 0.  *5,000 + 60 + 8 = 5,068* Understand and read 4-digit numbers on a number line. 
Choosing mental methods where appropriate	Use unitising and known facts to support mental calculations. *Make 1,405 from place value equipment.* *Add 2,000.* *Now add the 1,000s.* *1 thousand + 2 thousands = 3 thousands* *1,405 + 2,000 = 3,405*	Use unitising and known facts to support mental calculations.  *I can add the 100s mentally.* *200 + 300 = 500* *So, 4,256 + 300 = 4,556*	Use unitising and known facts to support mental calculations. *4,256 + 300 = ?* *2 + 3 = 5* *200 + 300 = 500* *4,256 + 300 = 4,556*

<table>
<tr>
<td>Column addition with exchange</td>
<td>Use place value equipment on a place value grid to organise thinking.

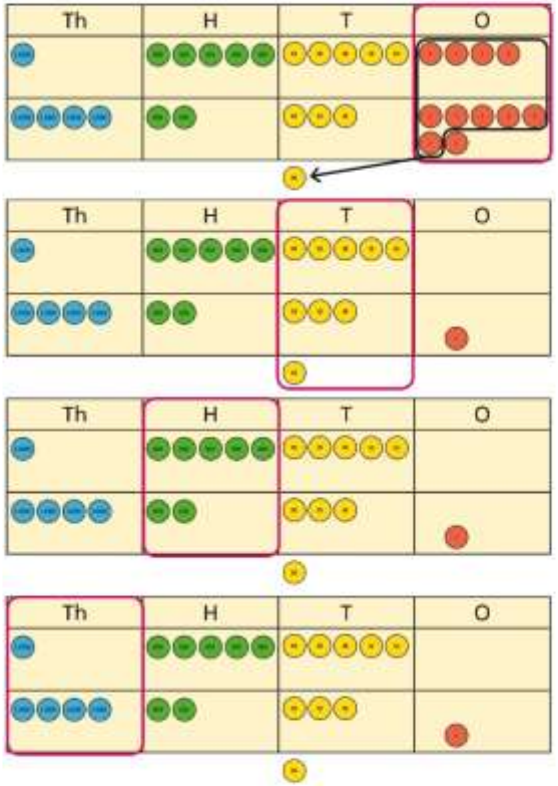
Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers.

Use equipment.to show 1,905 + 775.

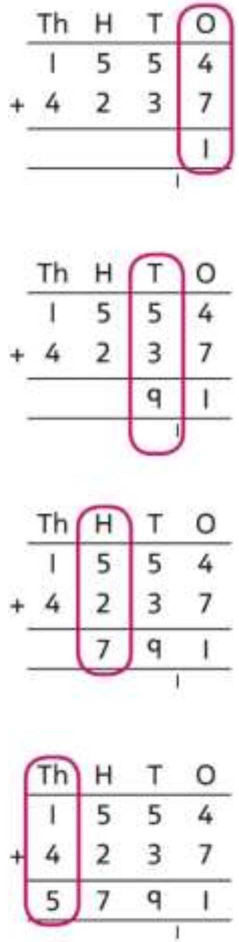


Why have only three columns been used for the second row? Why is the Thousands box empty?

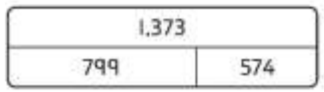
Which columns will total 10 or more?</td>
<td>Use place value equipment to model required exchanges.

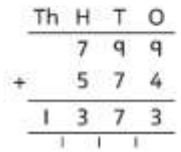


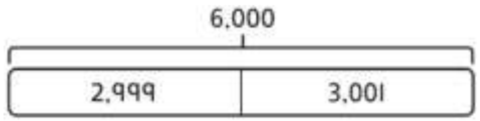
Include examples that exchange in more than one column.</td>
<td>Use a column method to add, including exchanges.



Include examples that exchange in more than one column.</td>
</tr>
</table>

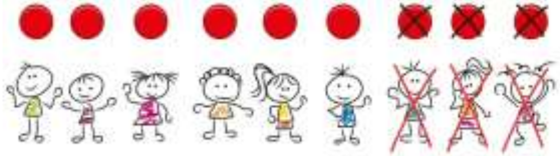
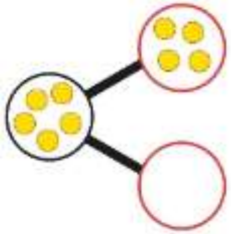
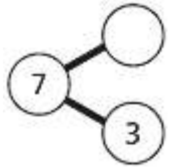
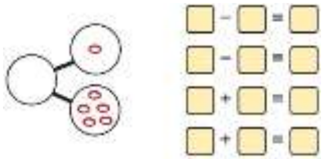
<table>
<tr>
<td>Representing additions and checking strategies</td>
<td></td>
<td>Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate.


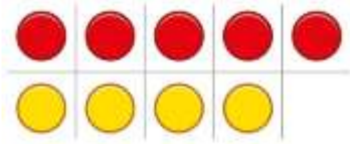
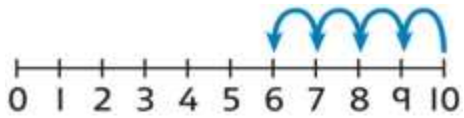
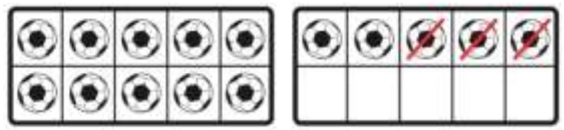


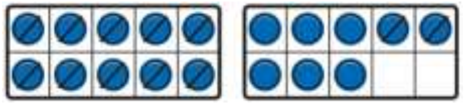
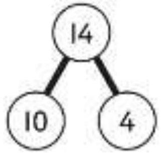
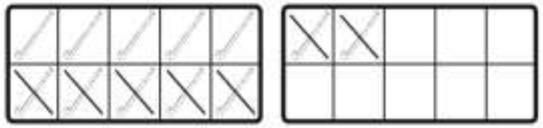
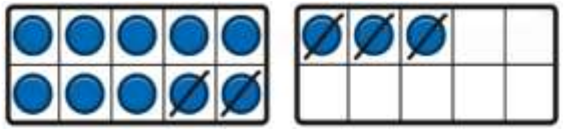
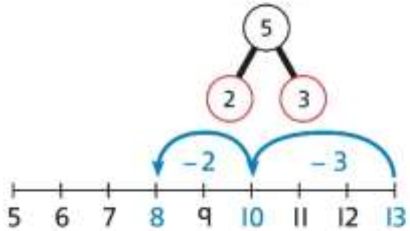
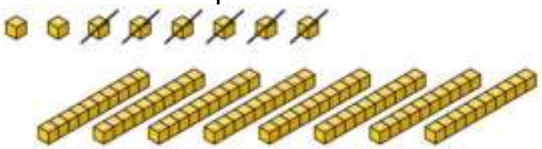
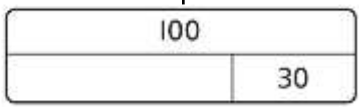
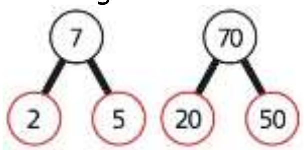
I chose to work out 574 + 800, then subtract 1.


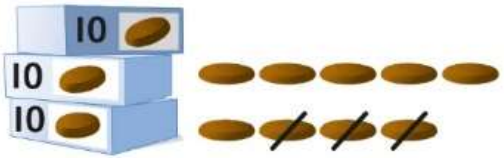
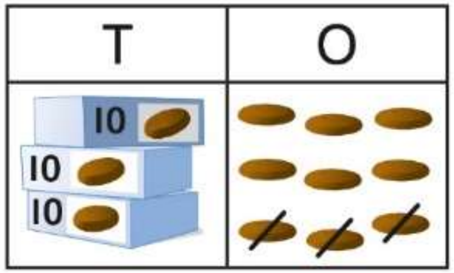
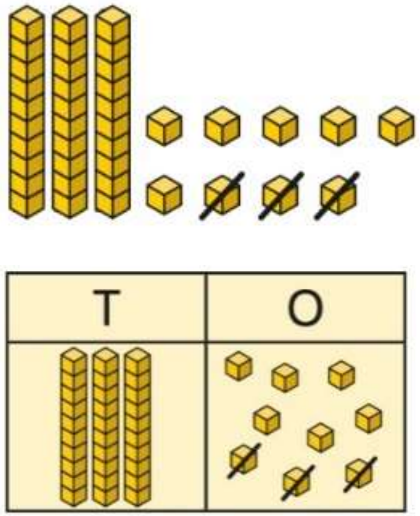
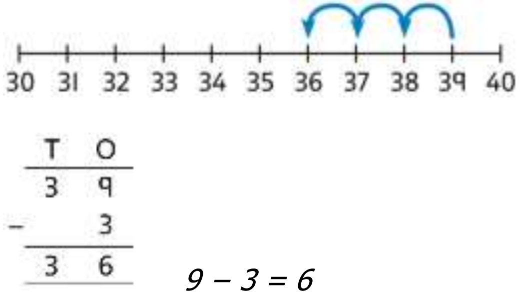
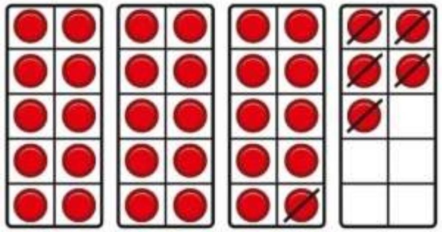
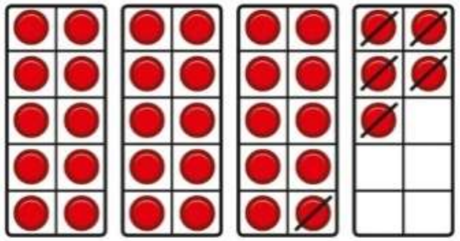
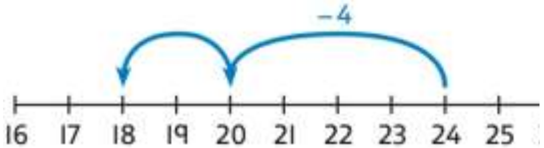
This is equivalent to 3,000 + 3,000.</td>
<td>Use rounding and estimating on a number line to check the reasonableness of an addition.

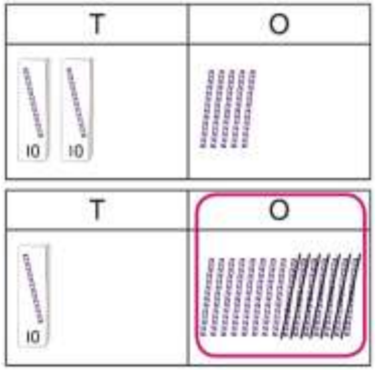
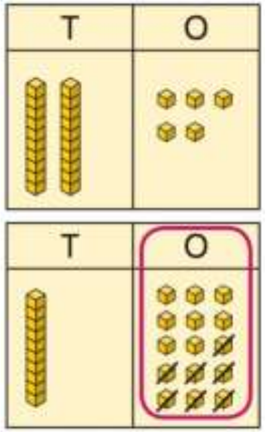
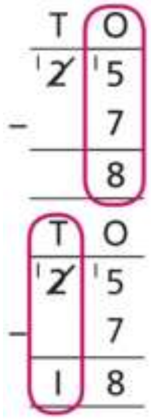
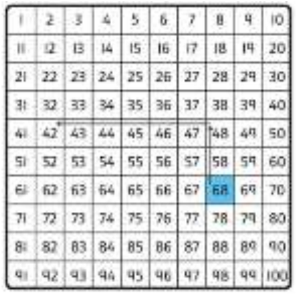
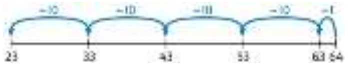
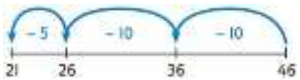

912 + 6,149 = ?
I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.</td>
</tr>
</table>

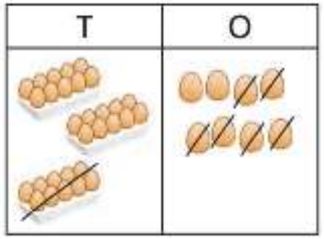
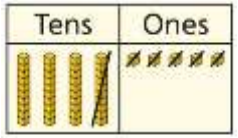
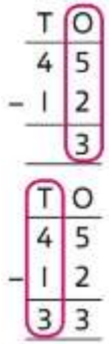
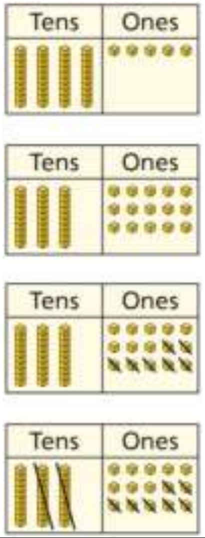
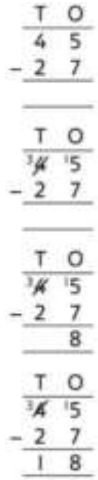
Subtraction			
	Concrete	Pictorial	Abstract
Counting back and taking away	Children arrange objects and remove to find how many are left.  *1 less than 6 is 5.* *6 subtract 1 is 5.*	Children draw and cross out or use counters to represent objects from a problem.  9 – ☐ = ☐ There are ☐ children left.	Children count back to take away and use a number line or number track to support the method.  *9 – 3 = 6*
Finding a missing part, given a whole and a part	Children separate a whole into parts and understand how one part can be found by subtraction.  *8 – 5 = ?*	Children represent a whole and a part and understand how to find the missing part by subtraction.  5 – 4 = ☐	Children use a part-whole model to support the subtraction to find a missing part.  *7 – 3 = ?* Children develop an understanding of the relationship between addition and subtraction facts in a part-whole model. 

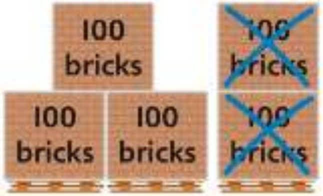
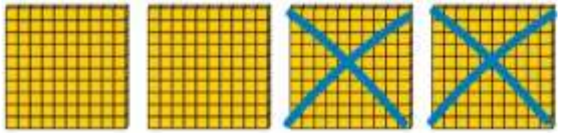
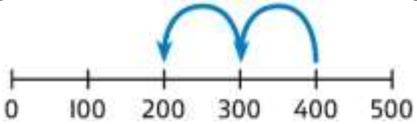
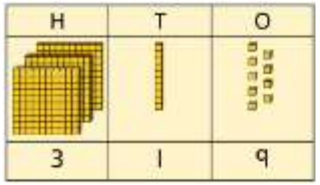
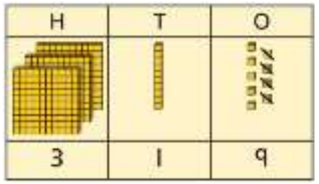
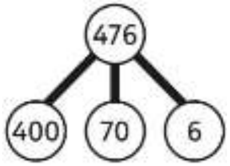
Finding the difference	Arrange two groups so that the difference between the groups can be worked out.  *8 is 2 more than 6.* *6 is 2 less than 8.* *The difference between 8 and 6 is 2.*	Represent objects using sketches or counters to support finding the difference.  *5 − 4 = 1* *The difference between 5 and 4 is 1.*	Children understand 'find the difference' as subtraction.  *10 − 4 = 6* *The difference between 10 and 6 is 4.*
Subtraction within 20	Understand when and how to subtract 1s efficiently. Use a bead string to subtract 1s efficiently.  *5 − 3 = 2* *15 − 3 = 12*	Understand when and how to subtract 1s efficiently.  *5 − 3 = 2* *15 − 3 = 12*	Understand how to use knowledge of bonds within 10 to subtract efficiently. *5 − 3 = 2* *15 − 3 = 12*

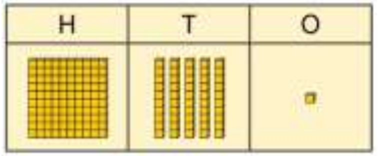
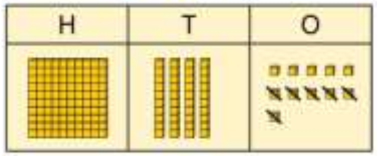
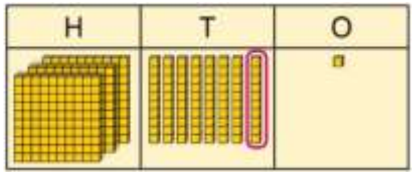
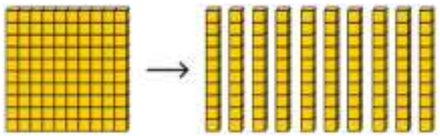
Subtracting 10s and 1s	For example: 18 – 12 Subtract 12 by first subtracting the 10, then the remaining 2.  *First subtract the 10, then take away 2.*	For example: 18 – 12 Use ten frames to represent the efficient method of subtracting 12.  *First subtract the 10, then subtract 2.*	Use a part-whole model to support the calculation.  *19 – 14* *19 – 10 = 9* *9 – 4 = 5* *So, 19 – 14 = 5*
Subtraction bridging 10 using number bonds	For example: 12 – 7 Arrange objects into a 10 and some 1s, then decide on how to split the 7 into parts.  *7 is 2 and 5, so I take away the 2 and then the 5.*	Represent the use of bonds using ten frames.  *For 13 – 5, I take away 3 to make 10, then take away 2 to make 8.*	Use a number line and a part-whole model to support the method. *13 – 5* 
Subtracting multiples of 10	Use known number bonds and unitising to subtract multiples of 10.  *8 subtract 6 is 2.* *So, 8 tens subtract 6 tens is 2 tens.*	Use known number bonds and unitising to subtract multiples of 10.  *10 – 3 = 7* *So, 10 tens subtract 3 tens is 7 tens.*	Use known number bonds and unitising to subtract multiples of 10.  *7 tens subtract 5 tens is 2 tens.* *70 – 50 = 20*

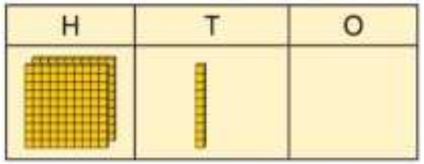
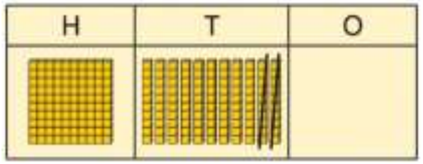
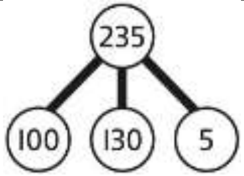
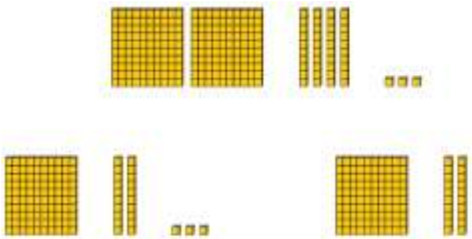
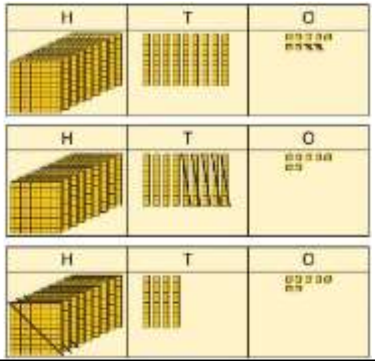
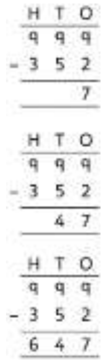
Subtracting a single-digit number	Subtract the 1s. This may be done in or out of a place value grid.   	Subtract the 1s. This may be done in or out of a place value grid.  	Subtract the 1s. Understand the link between counting back and subtracting the 1s using known bonds.  *9 – 3 = 6* *39 – 3 = 36*
Subtracting a single-digit number bridging 10	Bridge 10 by using known bonds.  *35 – 6* *I took away 5 counters, then 1 more.*	Bridge 10 by using known bonds.  *35 – 6* *First, I will subtract 5, then 1.*	Bridge 10 by using known bonds.  *24 – 6 = ?* *24 – 4 – 2 = ?*
Subtracting a single-digit	Exchange 1 ten for 10 ones. This may be done in or out of a place value grid.	Exchange 1 ten for 10 ones.	Exchange 1 ten for 10 ones.

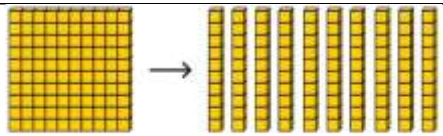
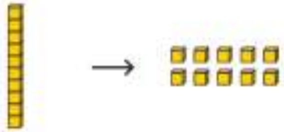
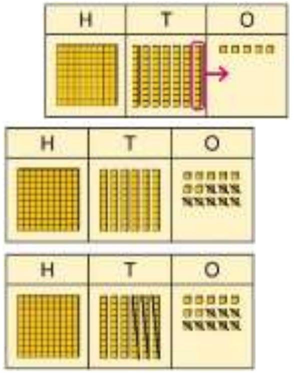
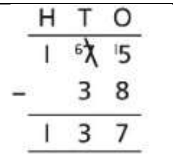
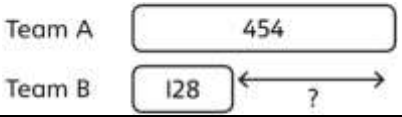
number using exchange			 *25 − 7 = 18*
Subtracting a 2-digit number	Subtract by taking away.  *61 − 18* *I took away 1 ten and 8 ones.*	Subtract the 10s and the 1s. This can be represented on a 100 square. 	Subtract the 10s and the 1s. This can be represented on a number line.  *64 − 41 = ?* *64 − 1 = 63* *63 − 40 = 23* *64 − 41 = 23*  *46 − 20 = 26* *26 − 5 = 21* *46 − 25 = 21*
Subtracting a 2-digit number using	Subtract the 1s. Then subtract the 10s. This may be done in or out of a place value grid.	Subtract the 1s. Then subtract the 10s.	Using column subtraction, subtract the 1s. Then subtract the 10s.

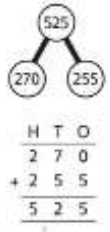
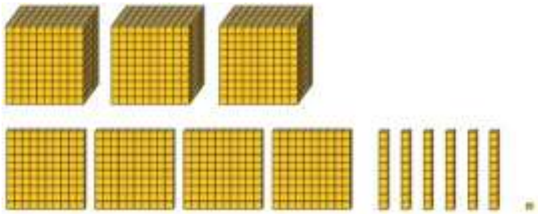
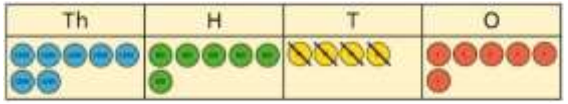
place value and columns	 *38 − 16 = 22*		
Subtracting a 2-digit number with exchange		Exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s. 	Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s. 
Subtracting 100s	Use known facts and unitising to subtract multiples of 100.	Use known facts and unitising to subtract multiples of 100.	Understand the link with counting back in 100s.

	 *5 – 2 = 3* *500 – 200 = 300*	 *4 – 2 = 2* *400 – 200 = 200*	 *400 – 200 = 200* Use known facts and unitising as efficient and accurate methods. *I know that 7 – 4 = 3. Therefore, I know that 700 – 400 = 300.*
3-digit number – 1s, no exchange	Use number bonds to subtract the 1s.  *214 – 3 = ?*  *4 – 3 = 1* *214 – 3 = 211*	Use number bonds to subtract the 1s.  *319 – 4 = ?*  *9 – 4 = 5* *319 – 4 = 315*	Understand the link with counting back using a number line. Use known number bonds to calculate mentally. *476 – 4 = ?*  *6 – 4 = 2* *476 – 4 = 472*
3-digit number – 1s, exchange or bridging required	Understand why an exchange is necessary by exploring why 1 ten must be exchanged. Use place value equipment.	Represent the required exchange on a place value grid. *151 – 6 = ?*	Calculate mentally by using known bonds. *151 – 6 = ?* *151 – 1 – 5 = 145*

		 	
3-digit number – 10s, no exchange	Subtract the 10s using known bonds.  *381 – 10 = ?* *8 tens with 1 removed is 7 tens.* *381 – 10 = 371*	Subtract the 10s using known bonds.  *8 tens – 1 ten = 7 tens* *381 – 10 = 371*	Use known bonds to subtract the 10s mentally. *372 – 50 = ?* *70 – 50 = 20* *So, 372 – 50 = 322*
3-digit number – 10s, exchange or bridging required	Use equipment to understand the exchange of 1 hundred for 10 tens. 	Represent the exchange on a place value grid using equipment. *210 – 20 = ?*	Understand the link with counting back on a number line. Use flexible partitioning to support the calculation. *235 – 60 = ?*

		 *I need to exchange 1 hundred for 10 tens, to help subtract 2 tens.*  *210 − 20 = 190*	 *235 = 100 + 130 + 5* *235 − 60 = 100 + 70 + 5* *= 175*
3-digit number – up to 3-digit number	Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away. 	Represent the calculation on a place value grid. 	Use column subtraction to calculate accurately and efficiently. 
3-digit number – up to 3-digit number, exchange required	Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones.	*Model the required exchange on a place value grid.* *175 − 38 = ?* *I need to subtract 8 ones, so I will exchange a ten for 10 ones.*	Use column subtraction to work accurately and efficiently.

	 	 	 175 – 38 = 137 If the subtraction is a 3-digit number subtract a 2-digit number, children should understand how the recording relates to the place value, and so how to line up the digits correctly. Children should also understand how to exchange in calculations where there is a zero in the 10s column. 
Representing subtraction problems		Use bar models to represent subtractions. ‘Find the difference’ is represented as two bars for comparison.  	Children use alternative representations to check calculations and choose efficient methods. Children use inverse operations to check additions and subtractions. The part-whole model supports understanding.

		Bar models can also be used to show that a part must be taken away from the whole.	*I have completed this subtraction.* *525 − 270 = 255* *I will check using addition.* 
Choosing mental methods where appropriate	Use place value equipment to justify mental methods.  *What number will be left if we take away 300?*	Use place value grids to support mental methods where appropriate.  *7,646 − 40 = 7,606*	Use knowledge of place value and unitising to subtract mentally where appropriate. *3,501 − 2,000* *3 thousands − 2 thousands = 1 thousand* *3,501 − 2,000 = 1,501*
Column subtraction with exchange	Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary.	Represent place value equipment on a place value grid to subtract, including exchanges where needed.	Use column subtraction, with understanding of the place value of any exchange required.

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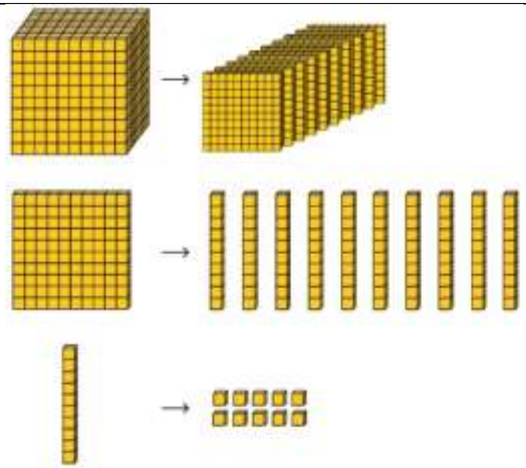
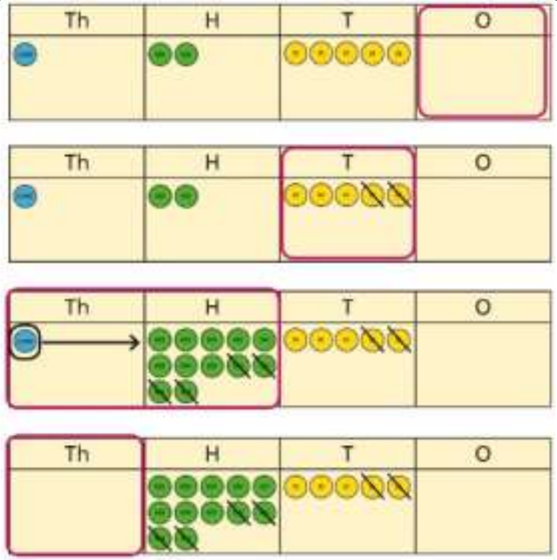
<td></td>
<td>
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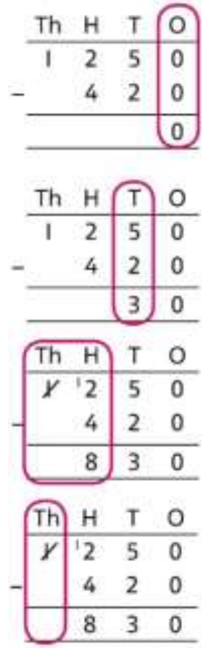
<td>
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<tr>
<td>Column subtraction with exchange across more than one column</td>
<td>Understand why two exchanges may be necessary.

2,502 − 243 = ?</td>
<td>Make exchanges across more than one column where there is a zero as a place holder.

2,502 − 243 = ?</td>
<td>Make exchanges across more than one column where there is a zero as a place holder.

2,502 − 243 = ?</td>
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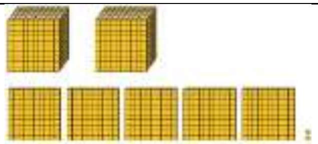
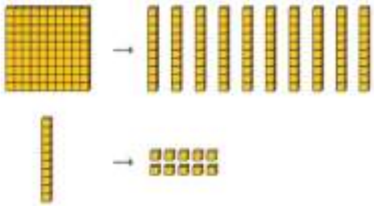
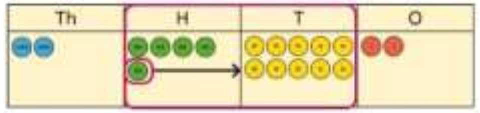
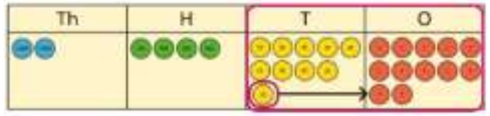
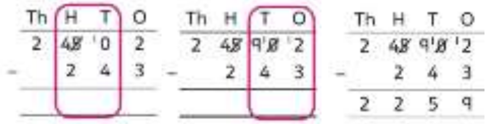
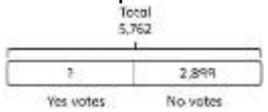
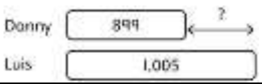
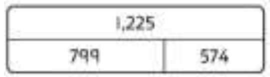
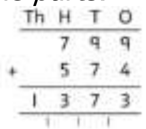
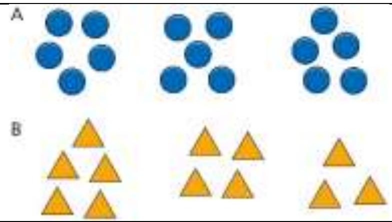
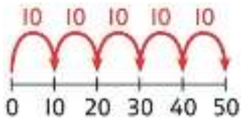
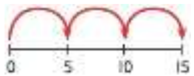
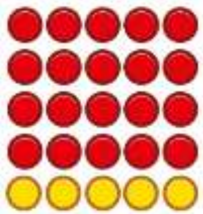
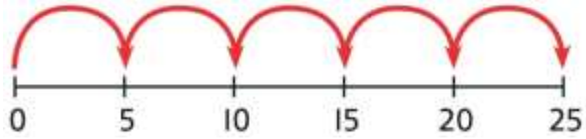
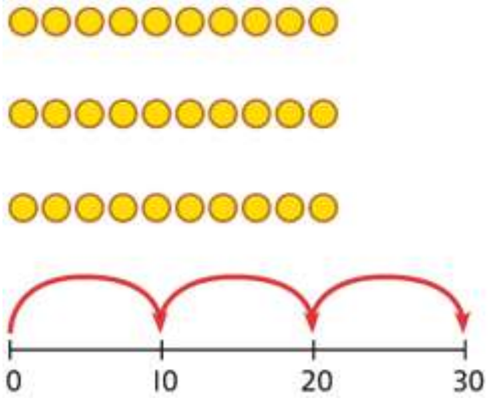
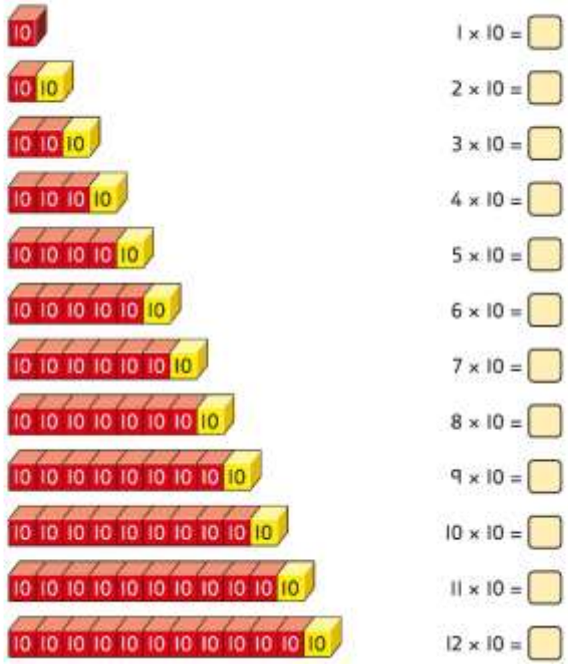
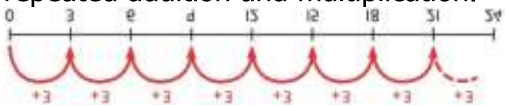
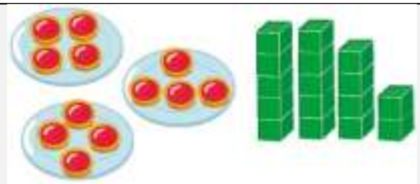
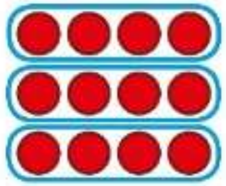
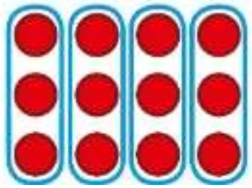
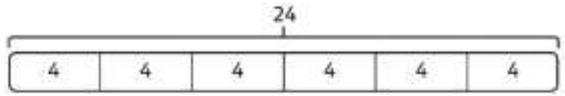
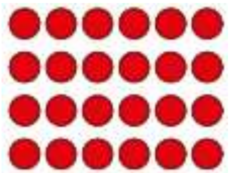
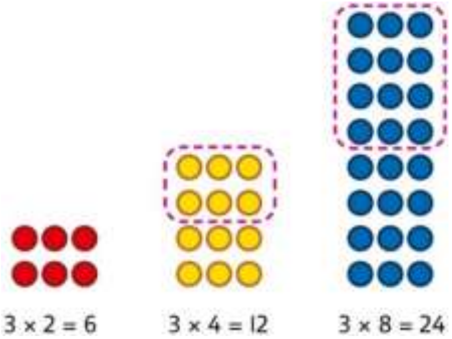
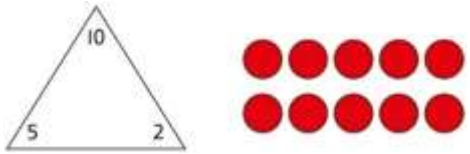
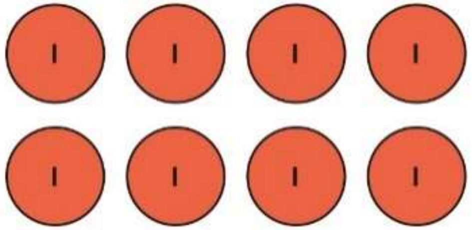
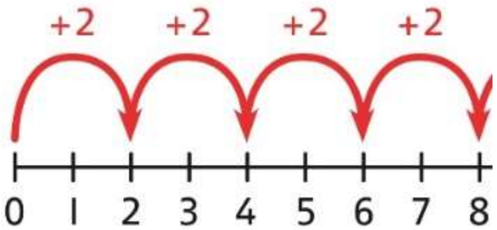
	 *I need to exchange a 10 for some 1s, but there are not any 10s here.* 	  	
Representing subtractions and checking strategies		Use bar models to represent subtractions where a part needs to be calculated.  *I can work out the total number of Yes votes using 5,762 – 2,899.* Bar models can also represent 'find the difference' as a subtraction problem.  	Use inverse operations to check subtractions. *I calculated 1,225 – 799 = 574.* *I will check by adding the parts.*   *The parts do not add to make 1,225.* *I must have made a mistake.*
		Multiplication	
	Concrete	Pictorial	Abstract
Recognising and making equal groups	Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal.	Children draw and represent equal and unequal groups.	*Three equal groups of 4.* *Four equal groups of 3.*

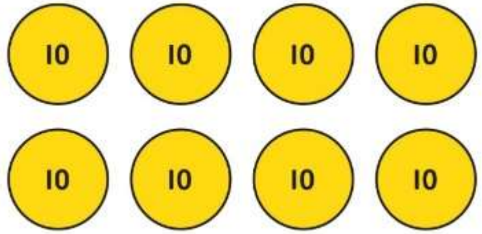
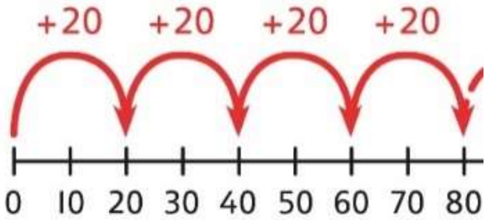
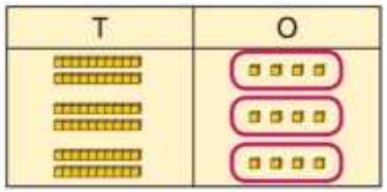
			
Finding the total of equal groups by counting in 2s, 5s and 10s	 There are 5 pens in each pack … 5…10…15…20…25…30…35…40 …	**Finding the total of equal groups by counting in 2s, 5s and 10s** 100 squares and ten frames support counting in 2s, 5s and 10s.  	**Finding the total of equal groups by counting in 2s, 5s and 10s** Use a number line to support repeated addition through counting in 2s, 5s and 10s. 
Equal groups and repeated addition	Recognise equal groups and write as repeated addition and as multiplication.  *3 groups of 5 chairs* *15 chairs altogether*	Recognise equal groups using standard objects such as counters and write as repeated addition and multiplication.  *3 groups of 5* *15 in total*	Use a number line and write as repeated addition and as multiplication.  *5 + 5 + 5 = 15* *3 × 5 = 15*

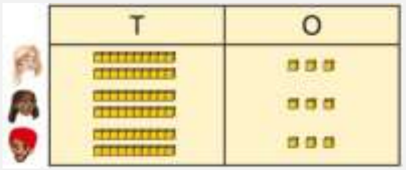
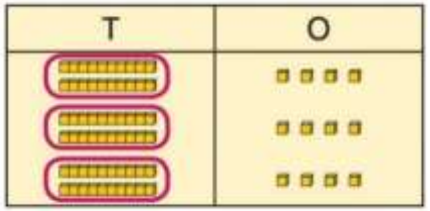
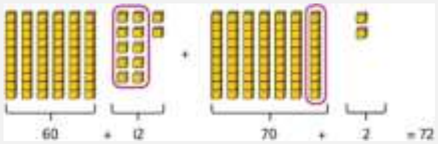
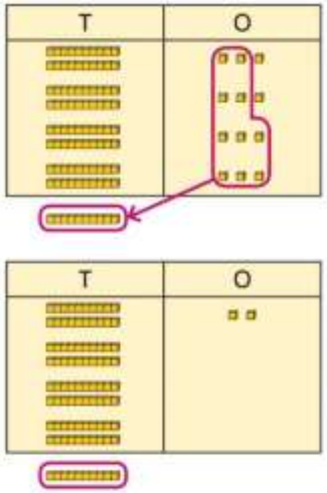
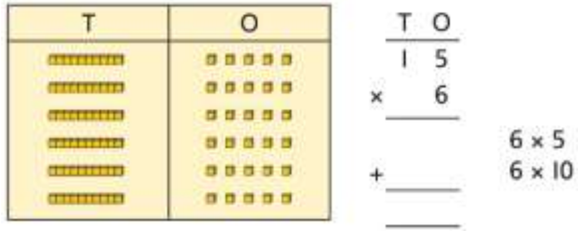
Using arrays to represent multiplication and support understanding	Understand the relationship between arrays, multiplication and repeated addition.  *4 groups of 5*	Understand the relationship between arrays, multiplication and repeated addition.  *4 groups of 5 … 5 groups of 5*	Understand the relationship between arrays, multiplication and repeated addition.  *5 × 5 = 25*
Understanding commutativity	Use arrays to visualise commutativity.  *I can see 6 groups of 3.* *I can see 3 groups of 6.*	Form arrays using counters to visualise commutativity. Rotate the array to show that orientation does not change the multiplication.  *This is 2 groups of 6 and also 6 groups of 2.*	Use arrays to visualise commutativity.   *4 + 4 + 4 + 4 + 4 = 20* *5 + 5 + 5 + 5 = 20* *4 × 5 = 20 and 5 × 4 = 20*
Learning ×2, ×5 and ×10 table facts	Develop an understanding of how to unitise groups of 2, 5	Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.	Understand how the times-tables increase and contain patterns.

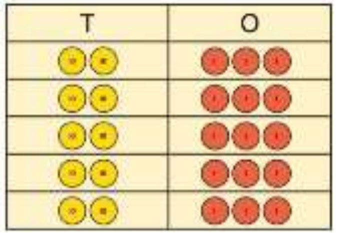
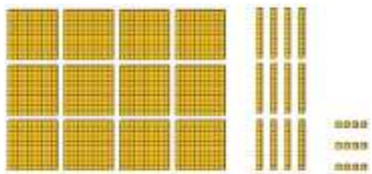
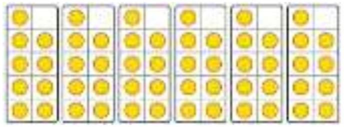
	and 10 and learn corresponding times-table facts.  *3 groups of 10 … 10, 20, 30* *3 × 10 = 30*	 *10 + 10 + 10 = 30* *3 × 10 = 30*	 *5 × 10 = 50* *6 × 10 = 60*
Understanding equal grouping and repeated addition	Children continue to build understanding of equal groups and the relationship with repeated addition. They recognise both examples and non-examples using objects.	Children recognise that arrays demonstrate commutativity.	Children understand the link between repeated addition and multiplication. 

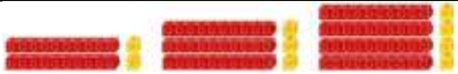
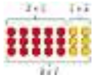
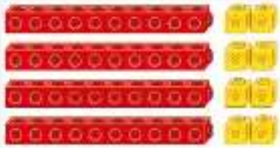
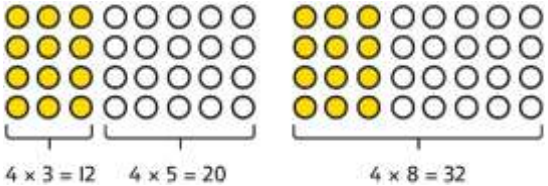
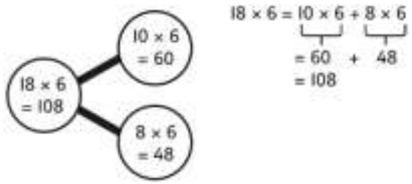
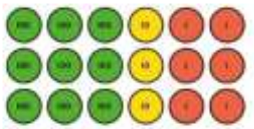
	 Children recognise that arrays can be used to model commutative multiplications.  *I can see 3 groups of 8.* *I can see 8 groups of 3.*	  *This is 3 groups of 4.* *This is 4 groups of 3.*	*8 groups of 3 is 24.* *3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24* *8 × 3 = 24* A bar model may represent multiplications as equal groups.  *6 × 4 = 24*
Using commutativity to support understanding of the times-tables	Understand how to use times-tables facts flexibly.   *There are 6 groups of 4 pens.*	Understand how times-table facts relate to commutativity.  *6 × 4 = 24* *4 × 6 = 24*	Understand how times-table facts relate to commutativity. *I need to work out 4 groups of 7.* *I know that 7 × 4 = 28* *so, I know that* *4 groups of 7 = 28* *and*

	There are 4 groups of 6 bread rolls. *I can use 6 × 4 = 24 to work out both totals.*		*7 groups of 4 = 28.*
Understanding and using ×3, ×2, ×4 and ×8 tables.	Children learn the times-tables as 'groups of', but apply their knowledge of commutativity.  *I can use the ×3 table to work out how many keys.* *I can also use the ×3 table to work out how many batteries.*	Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling. 	Children understand the relationship between related multiplication and division facts in known times-tables.  *2 × 5 = 10* *5 × 2 = 10* *10 ÷ 5 = 2* *10 ÷ 2 = 5*
Using known facts to multiply 10s, for example 3 × 40	Explore the relationship between known times-tables and multiples of 10 using place value equipment. *Make 4 groups of 3 ones.*  *Make 4 groups of 3 tens.* 	Understand how unitising 10s supports multiplying by multiples of 10. 	Understand how to use known times-tables to multiply multiples of 10. 

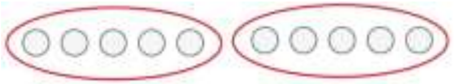
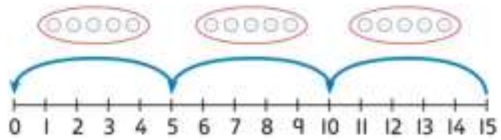
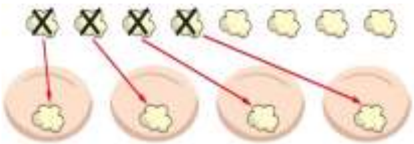
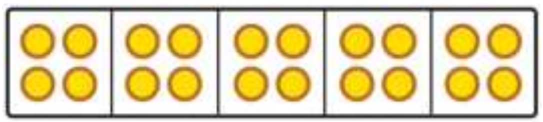
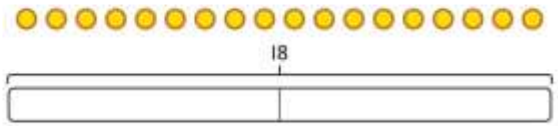
	What is the same? *What is different?*	 *4 groups of 2 ones is 8 ones.* *4 groups of 2 tens is 8 tens.* *4 × 2 = 8* *4 × 20 = 80*	 *4 × 2 = 8* *4 × 20 = 80*
Multiplying a 2-digit number by a 1-digit number	Understand how to link partitioning a 2-digit number with multiplying. *Each person has 23 flowers.* *Each person has 2 tens and 3 ones.*  *There are 3 groups of 2 tens.*	Use place value to support how partitioning is linked with multiplying by a 2-digit number. *3 × 24 = ?*  *3 × 4 = 12*	Use addition to complete multiplications of 2-digit numbers by a 1-digit number. *4 × 13 = ?* *4 × 3 = 12* *4 × 10 = 40* *12 + 40 = 52* *4 × 13 = 52*

	There are 3 groups of 3 ones. Use place value equipment to model the multiplication context.  *There are 3 groups of 3 ones.* *There are 3 groups of 2 tens.*	 *3 × 20 = 60* *60 + 12 = 72* *3 × 24 = 72*	
Multiplying a 2-digit number by a 1-digit number, expanded column method	Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications. *3 × 24 = ?* *3 × 20 = 60* *3 × 4 = 12*  *3 × 24 = 60 + 12* *3 × 24 = 70 + 2* *3 × 24 = 72*	Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s. *4 × 23 = ?*  *4 × 23 = 92*	Children may write calculations in expanded column form, but must understand the link with place value and exchange. Children are encouraged to write the expanded parts of the calculation separately.  *5 × 28 = ?*

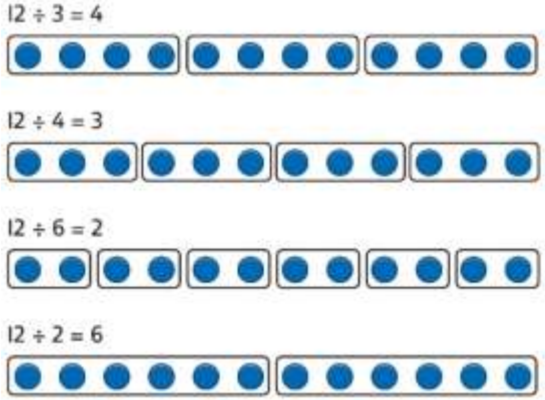
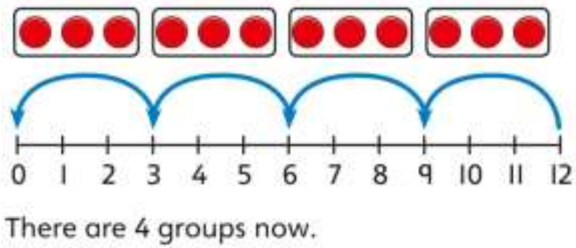
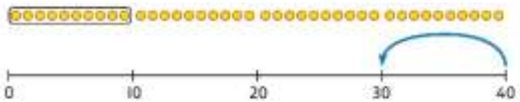
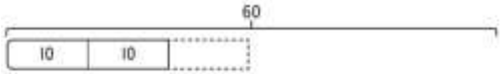
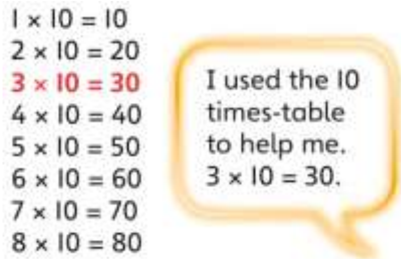
		 *5 × 23 = ?* *5 × 3 = 15* *5 × 20 = 100* *5 × 23 = 115*	T O 2 8 × 5 4 0 5 × 8 1 0 0 5 × 20 1 4 0
Multiplying by multiples of 10 and 100	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  *3 groups of 4 ones is 12 ones.* *3 groups of 4 tens is 12 tens.* *3 groups of 4 hundreds is 12 hundreds.*	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  *3 × 4 = 12* *3 × 40 = 120* *3 × 400 = 1,200*	Use known facts and understanding of place value and commutativity to multiply mentally. *4 × 7 = 28* *4 × 70 = 280* *40 × 7 = 280* *4 × 700 = 2,800* *400 × 7 = 2,800*
Understanding times-tables up to 12 × 12	Understand the special cases of multiplying by 1 and 0.  *5 × 1 = 5*	Represent the relationship between the ×9 table and the ×10 table.  Represent the ×11 table and ×12 tables in relation to the ×10 table.	Understand how times-tables relate to counting patterns. Understand links between the ×3 table, ×6 table and ×9 table *5 × 6 is double 5 × 3* ×5 table and ×6 table

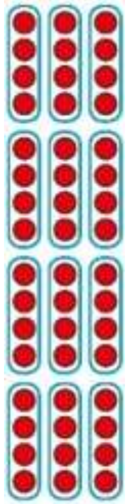
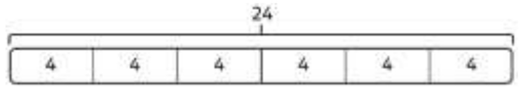
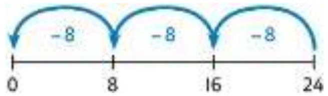
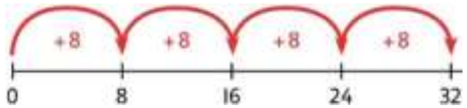
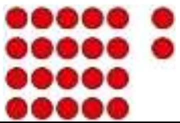
	5 × 0 = 0	 *2 × 11 = 20 + 2* *3 × 11 = 30 + 3* *4 × 11 = 40 + 4*  *4 × 12 = 40 + 8*	*I know that 7 × 5 = 35* *so I know that 7 × 6 = 35 + 7.* ×5 table and ×7 table *3 × 7 = 3 × 5 + 3 × 2*  ×9 table and ×10 table *6 × 10 = 60 so 6 × 9 = 60 − 6*
Understanding and using partitioning in multiplication	Make multiplications by partitioning. *4 × 12 is 4 groups of 10 and 4 groups of 2.*  *4 × 12 = 40 + 8*	Understand how multiplication and partitioning are related through addition.  *4 × 3 = 12* *4 × 5 = 20* *12 + 20 = 32* *4 × 8 = 32*	Use partitioning to multiply 2-digit numbers by a single digit. *18 × 6 = ?*  *18 × 6 = 10 × 6 + 8 × 6* *= 60 + 48* *= 108*
Column multiplication for 2- and 3-digit numbers multiplied by a single digit	Use place value equipment to make multiplications. *Make 4 × 136 using equipment.*  *I can work out how many 1s, 10s and 100s.*	Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.  $\begin{array}{r} 312 \\ \times \quad 3 \\ \hline 936 \end{array}$	Use the formal column method for up to 3-digit numbers multiplied by a single digit. $\begin{array}{r} 312 \\ \times \quad 3 \\ \hline 936 \end{array}$ Understand how the expanded column method is related to the formal column method and understand how any exchanges

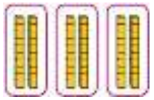
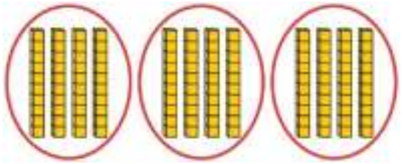
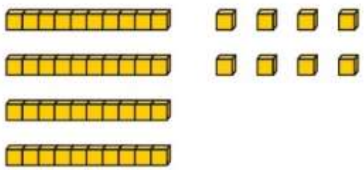
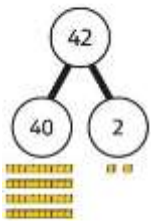
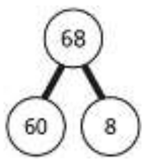
	There are 4 × 6 ones… 24 ones *There are 4 × 3 tens … 12 tens* *There are 4 × 1 hundreds … 4 hundreds* *24 + 120 + 400 = 544*		are related to place value at each stage of the calculation. $\begin{array}{r} 23 \\ \times\quad 5 \\ \hline 15 \\ 100 \\ \hline 115 \\ \hline \end{array}$ $\begin{array}{r} 23 \\ \times\quad 5 \\ \hline 115 \\ \hline {\scriptstyle 1}\ \ \end{array}$
Multiplying more than two numbers	Represent situations by multiplying three numbers together.  *Each sheet has 2 × 5 stickers.* *There are 3 sheets.* *There are 5 × 2 × 3 stickers in total.* $\underbrace{5 \times 2}_{10} \times 3 = 30$ $10 \times 3 = 30$	Understand that commutativity can be used to multiply in different orders.  *2 × 6 × 10 = 120* *12 × 10 = 120* *10 × 6 × 2 = 120* *60 × 2 = 120*	Use knowledge of factors to simplify some multiplications. *24 × 5 = 12 × 2 × 5* $12 \times \underbrace{2 \times 5}_{} =$ $12 \times 10 = 120$ So, 24 × 5 = 120

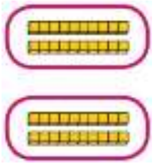
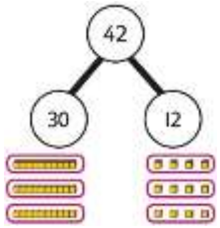
Division			
Grouping	Learn to make equal groups from a whole and find how many equal groups of a certain size can be made. Sort a whole set people and objects into equal groups. *There are 10 children altogether.* *There are 2 in each group.*  *There are 5 groups.*	Represent a whole and work out how many equal groups.  *There are 10 in total.* *There are 5 in each group.* *There are 2 groups.*	Children may relate this to counting back in steps of 2, 5 or 10. 
Sharing	Share a set of objects into equal parts and work out how many are in each part**.** 	Sketch or draw to represent sharing into equal parts. This may be related to fractions. 	*10 shared into 2 equal groups gives 5 in each group.*
Sharing equally	Start with a whole and share into equal parts, one at a time.	Represent the objects shared into equal parts using a bar model. 	Use a bar model to support understanding of the division. 

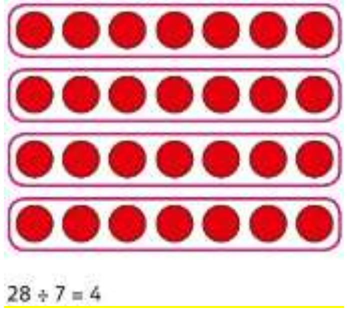
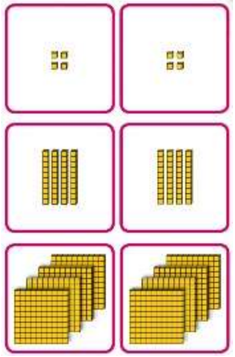
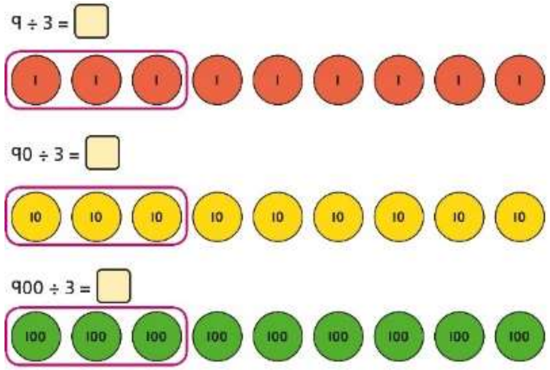
	 *12 shared equally between 2.* *They get 6 each.* Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person. Keep going until all the objects have been shared  *15 shared equally between 3.* *They get 5 each.*	*20 shared into 5 equal parts.* *There are 4 in each part.*	*18 ÷ 2 = 9*
Grouping equally	Understand how to make equal groups from a whole.  *8 divided into 4 equal groups.*	Understand the relationship between grouping and the division statements.	Understand how to relate division by grouping to repeated subtraction.

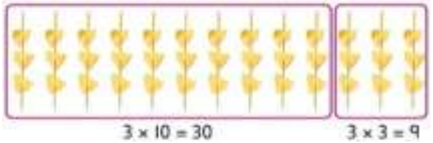
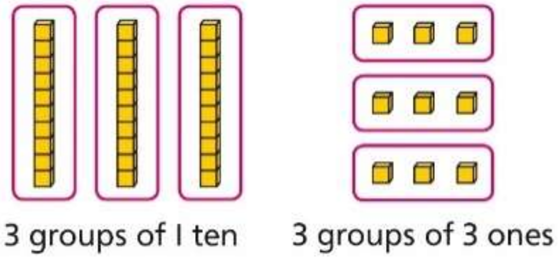
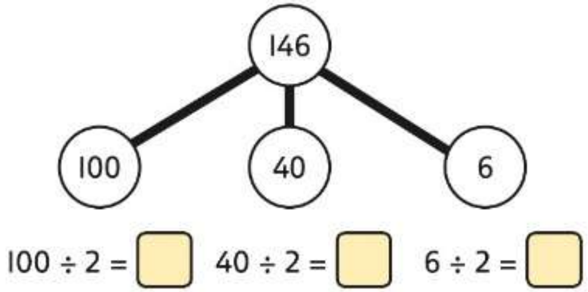
	There are 2 in each group.	 	 *12 divided into groups of 3.* *12 ÷ 3 = 4* *There are 4 groups.*
Using known times-tables to solve divisions	Understand the relationship between multiplication facts and division.  *4 groups of 5 cars is 20 cars in total.* *20 divided by 4 is 5.*	Link equal grouping with repeated subtraction and known times-table facts to support division.  *40 divided by 4 is 10.* Use a bar model to support understanding of the link between times-table knowledge and division.  	Relate times-table knowledge directly to division.  *I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.* *3 × 10 = 30 so 30 ÷ 10 = 3*

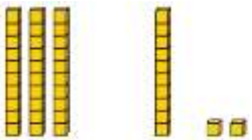
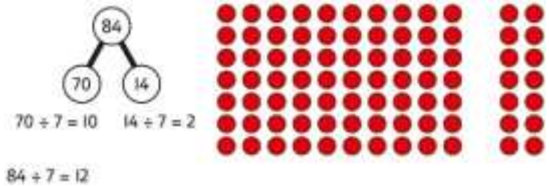
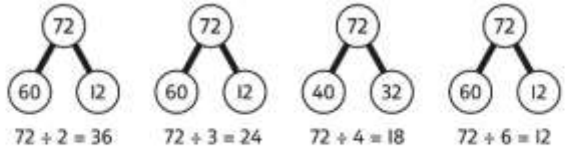
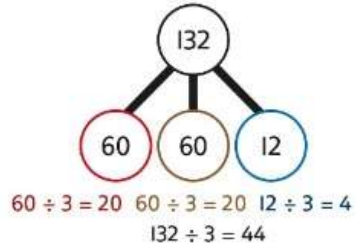
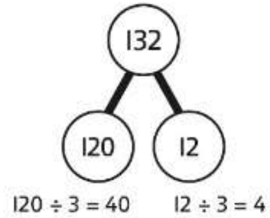
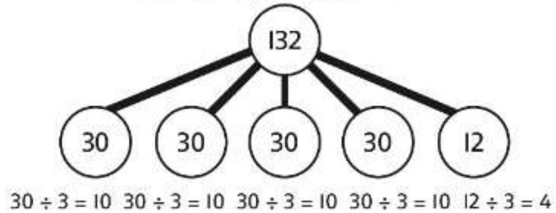
Using times-tables knowledge to divide	Use knowledge of known times-tables to calculate divisions.  *24 divided into groups of 8.* *There are 3 groups of 8.*	Use knowledge of known times-tables to calculate divisions.  48 ÷ 4 = 12 *48 divided into groups of 4.* *There are 12 groups.* *4 × 12 = 48* *48 ÷ 4 = 12*	Use knowledge of known times-tables to calculate divisions. *I need to work out 30 shared between 5.* *I know that 6 × 5 = 30* *so I know that 30 ÷ 5 = 6.* A bar model may represent the relationship between sharing and grouping.  *24 ÷ 4 = 6* *24 ÷ 6 = 4* Children understand how division is related to both repeated subtraction and repeated addition.  *24 ÷ 8 = 3*  *32 ÷ 8 = 4*
Understanding remainders	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.	Use images to explain remainders. 	Understand that the remainder is what cannot be shared equally from a set. *22 ÷ 5 = ?*

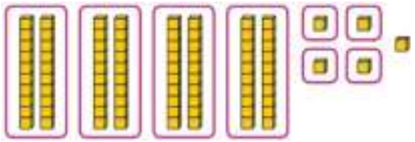
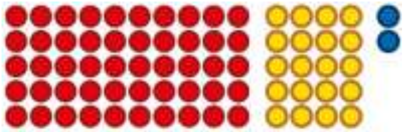
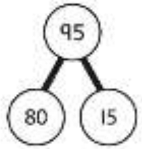
	 *There are 13 sticks in total.* *There are 3 groups of 4, with 1 remainder.*	*22 ÷ 5 = 4 remainder 2*	*3 × 5 = 15* *4 × 5 = 20* *5 × 5 = 25 … this is larger than 22* *So, 22 ÷ 5 = 4 remainder 2*
Using known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising. *Make 6 ones divided by 3.*  *Now make 6 tens divided by 3.*  *What is the same? What is different?*	Divide multiples of 10 by unitising.  *12 tens shared into 3 equal groups.* *4 tens in each group.*	Divide multiples of 10 by a single digit using known times-tables. *180 ÷ 3 = ?* *180 is 18 tens.* *18 divided by 3 is 6.* *18 tens divided by 3 is 6 tens.* *18 ÷ 3 = 6* *180 ÷ 3 = 60*
2-digit number divided by 1-digit number, no remainders	Children explore dividing 2-digit numbers by using place value equipment.  *48 ÷ 2 = ?*	Children explore which partitions support particular divisions. 	Children partition a number into 10s and 1s to divide where appropriate.  *60 ÷ 2 = 30* *8 ÷ 2 = 4* *30 + 4 = 34*

	First divide the 10s.  *Then divide the 1s.* 	*I need to partition 42 differently to divide by 3.*  *42 = 30 + 12* *42 ÷ 3 = 14*	*68 ÷ 2 = 34* Children partition flexibly to divide where appropriate. *42 ÷ 3 = ?* *42 = 40 + 2* *I need to partition 42 differently to divide by 3.* *42 = 30 + 12* *30 ÷ 3 = 10* *12 ÷ 3 = 4* *10 + 4 = 14* *42 ÷ 3 = 14*
2-digit number divided by 1-digit number, with remainders	Use place value equipment to understand the concept of remainder. *Make 29 from place value equipment.* *Share it into 2 equal groups.*  *There are two groups of 14 and 1 remainder.*	Use place value equipment to understand the concept of remainder in division. *29 ÷ 2 = ?*  *29 ÷ 2 = 14 remainder 1*	Partition to divide, understanding the remainder in context. *67 children try to make 5 equal lines.* *67 = 50 + 17* *50 ÷ 5 = 10* *17 ÷ 5 = 3 remainder 2* *67 ÷ 5 = 13 remainder 2* *There are 13 children in each line and 2 children left out.*
Understanding the relationship between	Use objects to explore families of multiplication and division facts.	Represent divisions using an array.	Understand families of related multiplication and division facts.

multiplication and division, including times-tables	 *4 × 6 = 24* *24 is 6 groups of 4.* *24 is 4 groups of 6.* *24 divided by 6 is 4.* *24 divided by 4 is 6.*		*I know that 5 × 7 = 35* *so I know all these facts:* *5 × 7 = 35* *7 × 5 = 35* *35 = 5 × 7* *35 = 7 × 5* *35 ÷ 5 = 7* *35 ÷ 7 = 5* *7 = 35 ÷ 5* *5 = 35 ÷ 7*
Dividing multiples of 10 and 100 by a single digit	Use place value equipment to understand how to use unitising to divide.  *8 ones divided into 2 equal groups* *4 ones in each group* *8 tens divided into 2 equal groups*	Represent divisions using place value equipment.  *9 ÷ 3 = 3* *9 tens divided by 3 is 3 tens.* *9 hundreds divided by 3 is 3 hundreds.*	Use known facts to divide 10s and 100s by a single digit. *15 ÷ 3 = 5* *150 ÷ 3 = 50* *1500 ÷ 3 = 500*

	4 tens in each group *8 hundreds divided into 2 equal groups* *4 hundreds in each group*		
Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s	Partition into 10s and 1s to divide where appropriate. *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate. *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate. *142 ÷ 2 = ?*  *100 ÷ 2 = 50* *40 ÷ 2 = 20* *6 ÷ 2 = 3* *50 + 20 + 3 = 73* *142 ÷ 2 = 73*

Dividing 2-digit and 3-digit numbers by a single digit, using flexible partitioning	Use place value equipment to explore why different partitions are needed. *42 ÷ 3 = ?* *I will split it into 30 and 12, so that I can divide by 3 more easily.* 	Represent how to partition flexibly where needed. *84 ÷ 7 = ?* *I will partition into 70 and 14 because I am dividing by 7.* 	Make decisions about appropriate partitioning based on the division required.  Understand that different partitions can be used to complete the same division.   
Understanding remainders	Use place value equipment to find remainders.	Represent the remainder as the part that cannot be shared equally.	Understand how partitioning can reveal remainders of divisions.

	85 shared into 4 equal groups *There are 24, and 1 that cannot be shared.* 	 *72 ÷ 5 = 14 remainder 2*	 *80 ÷ 4 = 20* *12 ÷ 4 = 3* *95 ÷ 4 = 23 remainder 3*